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Integration of products "By Parts"

We know that From differential Calculus of u, v_1 are two differentiable function of x

then $\frac{d}{dx}(uv_1) = u\frac{dv_1}{dx} + v_1\frac{du}{dx}$

Integrating both side.

$$\Rightarrow uv_1 = \int\left(u\frac{dv_1}{dx}\right)\cdot dx + \int\left(v_1\cdot\frac{du}{dx}\right)\cdot dx$$

$$\Rightarrow uv_1 - \int\left(v_1\cdot\frac{du}{dx}\right)\cdot dx = \int\left(u\cdot\frac{dv_1}{dx}\right)\cdot dx$$

let $\frac{dv_1}{dx} = v \Rightarrow v_1 = \int v\cdot dx$

$$\Rightarrow \int(uv)\cdot dx = u\int v\cdot dx - \int\left\{\left[\int v\cdot dx\right]\cdot\frac{du}{dx}\right\}\cdot dx$$

Thus $\Rightarrow$ Integral of product of two Functions

= 1.st Function (uncharged) * Integral of 2.nd
− Integral of {diff. Coeff of I.st * integral of 2.nd}

Formulas :-

(i) $\int x^n\cdot dx = \frac{x^{n+1}}{n+1}$, $n \neq -1$

(ii) $\int\frac{1}{x}\cdot dx = \log x$

(iii) $\int e^x\cdot dx = e^x$

(iv) $\int a^x\cdot dx = \frac{a^x}{\log a}$

(v) $\int \sin x\cdot dx = -\cos x$

(vi) $\int \cos x \cdot dx = \sin x$

(vii) $\int \tan x \cdot dx = \log \sec x$ or $-\log \cos x$.

(viii) $\int \cot x \cdot dx = \log \sin x$.

(ix) $\int \sec x \cdot dx = \log(\sec x + \tan x) = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

(x) $\int \operatorname{cosec} x \cdot dx = \log(\operatorname{cosec} x - \cot x)$

$= \log \tan \frac{x}{2}$.

(xi) $\int \frac{1}{a^2+x^2} \cdot dx = \frac{1}{a} \tan^{-1} x/a$

(xii) $\int \frac{1}{\sqrt{a^2-x^2}} \cdot dx = \sin^{-1} x/a$

(xiii) $\int \frac{1}{\sqrt{x^2-a^2}} \cdot dx = \cos^{-1} x/a$

(xiv) $\int \sqrt{a^2-x^2} \cdot dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$

(xv) $\int \sqrt{a^2+x^2} \cdot dx = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$

(xvi) $\int \sqrt{x^2-a^2} \cdot dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$

(xvii) $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1} x/a$

(xviii) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right)$

(xix) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right)$

(xx) $\int e^{ax} \cdot \sin bx \cdot dx$

$= \frac{e^{ax}}{a^2+b^2} (a \sin bx - b \cos bx)$

(xxi) $\int e^{ax} \cdot \cos bx \cdot dx = \frac{e^{ax}}{a^2+b^2} (a \cos bx + b \sin bx)$

$=\!=$.